

Edge and Total Edge Product Cordial Labeling of Some New Graphs

Chirag M. Barasara

Department of Mathematics

Hemchandracharya North Gujarat University

Patan - 384265, Gujarat, INDIA.

Abstract

In this paper we investigate edge product cordial labeling and total edge product cordial labeling for cycle with one chord, cycle with twin chords, triangular ladder graph and comb graph.

Keywords: Cordial graph, product cordial graph, edge product cordial graph, total edge product cordial labeling.

AMS Subject Classification(2010): 05C78.

1 Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with order p and size q . For all standard terminology and notation we follow Clark and Holton [2]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

For an extensive survey on graph labeling and bibliographic references we refer to Gallian [3].

In 1987, Cahit [1] introduced the cordial labeling as a weaker version of graceful and harmonious labelings. Some labeling schemes are also introduced with minor variations in cordial theme. In 2004, Sundaram *et al.* [6] have introduced product cordial labeling in which the absolute difference in cordial labeling is replaced by product of the vertex labels.

The edge analogue of product cordial labeling was introduced by Vaidya and Barasara [7] and they named it as edge product cordial labeling which is defined as follows.

Definition 1.2. For a graph $G = (V(G), E(G))$, an edge labeling function $f : E(G) \rightarrow \{0, 1\}$ induces a vertex labeling function $f : V(G) \rightarrow \{0, 1\}$ defined as $f(v) = \pi f(e_i)$ for $\{e_i \in E(G)/e_i$ is incident to $v\}$.

Now denoting the number of vertices of G having label i under f as $v_f(i)$ and the number of edges of G having label i under f as $e_f(i)$.

Then f is called *edge product cordial labeling* of graph G if $|v_f(0)v_f(1)| \leq 1$ and $|e_f(0)e_f(1)| \leq 1$. A graph G is called *edge product cordial* if it admits edge product cordial labeling.

In [8, 9, 10, 11, 12], Vaidya and Barasara have investigated several results related to edge product cordial labeling. Prajapati and Shah [4] have proved some results related to edge product cordial labeling in the context of duplication of some graph elements while Prajapati and Patal [5] have discussed edge product cordial labeling for some cycle related graphs.

The edge analogue of total product cordial labeling was introduced by Vaidya and Barasara [13] and they named it as total edge product cordial labeling which is defined as follows.

Definition 1.3. For a graph $G = (V(G), E(G))$, an edge labeling function $f : E(G) \rightarrow \{0, 1\}$ induces a vertex labeling function $f : V(G) \rightarrow \{0, 1\}$ defined as $f(v) = \pi f(e_i)$ for $\{e_i \in E(G) / e_i \text{ is incident to } v\}$.

Now denoting the number of vertices of G having label i under f as $v_f(i)$ and the number of edges of G having label i under f as $e_f(i)$.

Then f is called *total edge product cordial labeling* of graph G if $|(v_f(0) + e_f(0))(v_f(1) + e_f(1))| \leq 1$. A graph G is called *total edge product cordial* if it admits total edge product cordial labeling.

In [14], Vaidya and Barasara have discussed total edge product cordial labeling in the context of various graph operations.

Proposition 1.4. [13] Every edge product cordial graph of either even order or even size admits total edge product cordial labeling.

Definition 1.5. A *chord of cycle* C_n is an edge joining two non-adjacent vertices of cycle C_n .

Definition 1.6. Two chords of a cycle C_n are said to be *twin chords* if they form a triangle with an edge of cycle C_n .

Definition 1.7. The ladder graph L_n is defined as $P_2 \square P_n$.

Definition 1.8. The triangular ladder graph TL_n is obtain from ladder graph L_n by adding the edges $u_i v_{i+1}$ for $1 \leq i \leq n - 1$.

Definition 1.9. The *comb* $(P_n \odot K_1)$ is obtained by joining a pendant edge to each vertex of path P_n .

In this paper we have investigated edge and total edge product cordial labeling for cycle with one chord, cycle with twin chords, triangular ladder graph and comb graph.

2 Main Results

Theorem 2.1. Cycle C_n with one chord is an edge product cordial graph except when n is even and chord is joining vertices which are at diameter distance.

Proof: Let the graph G be the cycle with one chord. Let $v_1, v_2, v_3, \dots, v_n$ be the vertices and e_1, e_2, \dots, e_n be the edges of cycle C_n . To define $f : E(G) \rightarrow \{0, 1\}$, we consider following two cases.

Case 1: When n is odd.

Without loss of generality we assume that the chord is $e_{n+1} = v_1 v_i$ where $3 \leq i \leq \frac{n+1}{2}$.

$$\begin{aligned}
 f(e_i) &= 0; & 1 \leq i \leq \frac{n-1}{2}, \\
 f(e_{n+1}) &= 0, \\
 f(e_i) &= 1; & \frac{n+1}{2} \leq i \leq n.
 \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned}
 v_f(0) &= \frac{n+1}{2} \text{ and } v_f(1) = \frac{n-1}{2} \\
 e_f(0) &= e_f(1) = \frac{n+1}{2}
 \end{aligned}$$

Thus in this case we have $|v_f(0) - v_f(1)| = 1$ and $|e_f(0) - e_f(1)| = 0$.

Case 2: When n is even.

Subcase 1: If the chord is $e_{n+1} = v_1 v_{\frac{n}{2}+1}$.

In order to satisfy the edge condition for a graph to be edge product cordial it is essential to assign label 0 to at least $\frac{n}{2}$ edges out of $n+1$ edges. The edges with label 0 will give rise to at least $\frac{n}{2} + 1$ vertices with label 0 and at most $\frac{n}{2} - 1$ vertices with label 1 out of total n vertices. Therefore $|v_f(0) - v_f(1)| \geq 2$. Thus the vertex condition for a graph to be edge product cordial is violated.

Subcase 2: Without loss of generality we assume that the chord is $e_{n+1} = v_1 v_i$ where $3 \leq i \leq \frac{n}{2}$.

$$\begin{aligned}
 f(e_i) &= 0; & 1 \leq i \leq \frac{n}{2} - 1, \\
 f(e_{n+1}) &= 0, \\
 f(e_i) &= 1; & \frac{n}{2} \leq i \leq n.
 \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned}
 v_f(0) &= v_f(1) = \frac{n}{2} \\
 e_f(0) &= \frac{n}{2} \text{ and } e_f(1) = \frac{n}{2} + 1
 \end{aligned}$$

Thus in this case we have $|v_f(0) - v_f(1)| = 0$ and $|e_f(0) - e_f(1)| = 1$.

Hence, cycle C_n with one chord is an edge product cordial graph except when n is even and chord is joining vertices which are at diameter distance. ■

Illustration 2.2. The cycle C_7 with one chord and its edge product cordial labeling is shown in Figure 1.

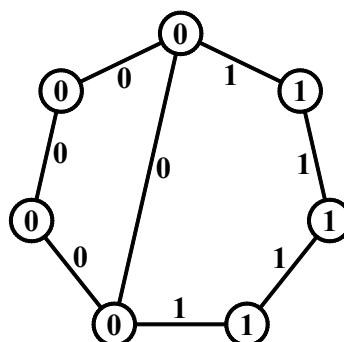


Figure 1

Theorem 2.3. Cycle C_n with one chord is a total edge product cordial graph.

Proof: Let the graph G be the cycle with one chord. Let $v_1, v_2, v_3, \dots, v_n$ be the vertices and e_1, e_2, \dots, e_n be the edges of cycle C_n . To define $f : E(G) \rightarrow \{0, 1\}$, we consider following two cases.

Case 1: When n is odd.

Here graph G is of even size and it is edge product cordial as proved in Theorem 2.1. Then by Proposition 1.4 the result holds.

Case 2: When n is even.

Subcase 1: If the chord is $e_{n+1} = v_1v_{\frac{n}{2}+1}$.

$$f(e_i) = 0; \quad 1 \leq i \leq \frac{n}{2},$$

$$f(e_i) = 1; \quad \frac{n}{2} + 1 \leq i \leq n + 1.$$

In view of the above defined labeling pattern we have

$$v_f(0) = \frac{n}{2} + 1 \text{ and } v_f(1) = \frac{n}{2} - 1$$

$$e_f(0) = \frac{n}{2} \text{ and } e_f(1) = \frac{n}{2} + 1$$

Thus in this case we have $|(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))| = 1$.

Subcase 2: Without loss of generality we assume that the chord is $e_{n+1} = v_1v_i$ where $3 \leq i \leq \frac{n}{2}$.

Here graph G is of even order and it is edge product cordial as proved in Theorem 2.1. Then by Proposition 1.4 the result holds.

Hence, cycle C_n with one chord is a total edge product cordial graph. ■

Illustration 2.4. The cycle C_6 with one chord and its total edge product cordial labeling is shown in Figure 2.

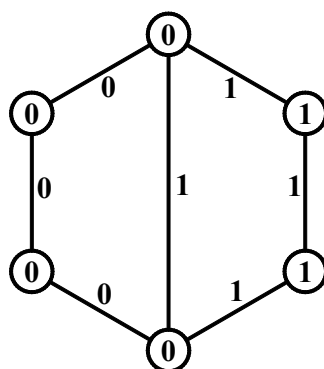


Figure 2

Theorem 2.5. Cycle C_n with twin chords is an edge product cordial graph except when n is even and a chord joining vertices which are at diameter distance.

Proof: Let the graph G be the cycle with twin chord. Let $v_1, v_2, v_3, \dots, v_n$ be the vertices and e_1, e_2, \dots, e_n be the edges of cycle C_n . To define $f : E(G) \rightarrow \{0, 1\}$, we consider following two cases.

Case 1: When n is odd.

Without loss of generality we assume that the twin chords are $e_{n+1} = v_1v_i$ and $e_{n+2} = v_1v_{i+1}$ where $3 \leq i \leq \frac{n+1}{2}$.

$$\begin{aligned} f(e_i) &= 0; & 1 \leq i \leq \frac{n-1}{2}, \\ f(e_{n+1}) &= 0, \\ f(e_i) &= 1; & \frac{n+1}{2} \leq i \leq n, \\ f(e_{n+2}) &= 1. \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned} v_f(0) &= \frac{n+1}{2} \text{ and } v_f(1) = \frac{n-1}{2} \\ e_f(0) &= \frac{n+1}{2} \text{ and } e_f(1) = \frac{n+3}{2} \end{aligned}$$

Thus in this case we have $|v_f(0) - v_f(1)| = 1$ and $|e_f(0) - e_f(1)| = 1$.

Case 2: When n is even.

Subcase 1: If the twin chords are $e_{n+1} = v_1v_{\frac{n}{2}}$ and $e_{n+2} = v_1v_{\frac{n}{2}+1}$.

In order to satisfy the edge condition for a graph to be edge product cordial it is essential to assign label 0 to at least $\frac{n}{2} + 1$ edges out of $n + 2$ edges. The edges with label 0 will give rise to at least $\frac{n}{2} + 1$ vertices with label 0 and at most $\frac{n}{2} - 1$ vertices with label 1 out of total n vertices. Therefore $|v_f(0) - v_f(1)| \geq 2$. Thus the vertex condition for a graph to be edge product cordial is violated.

Subcase 2: Without loss of generality we assume that the twin chords are $e_{n+1} = v_1v_i$ and $e_{n+2} = v_1v_{i+1}$ where $3 \leq i \leq \frac{n}{2} - 1$.

$$\begin{aligned} f(e_i) &= 0; & 1 \leq i \leq \frac{n}{2} - 1, \\ f(e_{n+1}) &= 0, \\ f(e_{n+2}) &= 0, \\ f(e_i) &= 1; & \frac{n}{2} \leq i \leq n. \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned} v_f(0) &= v_f(1) = \frac{n}{2} \\ e_f(0) &= e_f(1) = \frac{n}{2} + 1 \end{aligned}$$

Thus in this case we have $|v_f(0) - v_f(1)| = 0$ and $|e_f(0) - e_f(1)| = 0$.

Hence, cycle C_n with twin chords is an edge product cordial graph except when n is even and a chord joining vertices which are at diameter distance. ■

Illustration 2.6. The cycle C_9 with twin chord and its edge product cordial labeling is shown in Figure 3.

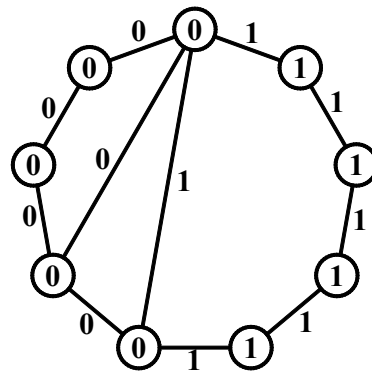


Figure 3

Theorem 2.7. Cycle C_n with twin chords is a total edge product cordial graph.

Proof: Let the graph G be the cycle with twin chord. Let $v_1, v_2, v_3, \dots, v_n$ be the vertices and e_1, e_2, \dots, e_n be the edges of cycle C_n . To define $f : E(G) \rightarrow \{0, 1\}$, we consider following two cases.

Case 1: When n is odd.

Without loss of generality we assume that the twin chords are $e_{n+1} = v_1v_i$ and $e_{n+2} = v_1v_{i+1}$ where $3 \leq i \leq \frac{n+1}{2}$.

$$\begin{aligned} f(e_i) &= 0; & 1 \leq i \leq \frac{n-1}{2}, \\ f(e_i) &= 1; & \frac{n+1}{2} \leq i \leq n, \\ f(e_{n+1}) &= 0, \\ f(e_{n+2}) &= 1. \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned} v_f(0) &= \frac{n+1}{2} \text{ and } v_f(1) = \frac{n-1}{2} \\ e_f(0) &= \frac{n+1}{2} \text{ and } e_f(1) = \frac{n+3}{2} \end{aligned}$$

Thus in this case we have $|(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))| = 0$.

Case 2: When n is even.

Subcase 1: If the twin chords are $e_{n+1} = v_1v_{\frac{n}{2}}$ and $e_{n+2} = v_1v_{\frac{n}{2}+1}$.

$$\begin{aligned} f(e_i) &= 1; & 1 \leq i \leq \frac{n}{2}, \\ f(e_i) &= 0; & \frac{n}{2} + 1 \leq i \leq n, \\ f(e_{n+1}) &= 1, \\ f(e_{n+2}) &= 1. \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned} v_f(0) &= \frac{n}{2} + 1 \text{ and } v_f(1) = \frac{n}{2} - 1 \\ e_f(0) &= \frac{n}{2} \text{ and } e_f(1) = \frac{n}{2} + 2 \end{aligned}$$

Thus in this case we have $|(v_f(0) + e_f(0)) - (v_f(1) - e_f(1))| = 0$.

Subcase 2: Without loss of generality we assume that the twin chords are $e_{n+1} = v_1v_i$ and $e_{n+2} = v_1v_{i+1}$ where $3 \leq i \leq \frac{n}{2} - 1$.

Here graph G is of even order and it is edge product cordial as proved in Theorem 2.5. Then by Proposition 1.4 the result holds.

Hence, cycle C_n with twin chords is a total edge product cordial graph. ■

Illustration 2.8. The cycle C_8 with twin chord and its total edge product cordial labeling is shown in Figure 4.

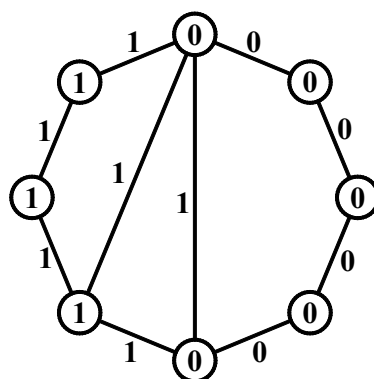


Figure 4

Theorem 2.9. The triangular ladder TL_n is not an edge product cordial graph.

Proof: The triangular ladder TL_n is of order $2n$ and size $4n - 3$. In order to satisfy the edge condition for a graph to be edge product cordial it is essential to assign label 0 to at least $2n - 2$ edges out of $4n - 3$ edges. The edges with label 0 will give rise to at least $n + 1$ vertices with label 0 and at most $n - 1$ vertices with label 1 out of total $2n$ vertices. Therefore $|v_f(0) - v_f(1)| \geq 2$. Thus the vertex condition for a graph to be edge product cordial is violated. Hence, the triangular ladder TL_n is not an edge product cordial graph. ■

Theorem 2.10. The triangular ladder TL_n is a total edge product cordial graph.

Proof: The triangular ladder TL_n is of order $2n$ and size $4n - 3$. Let u_1, u_2, \dots, u_n be the vertices corresponding to first path while v_1, v_2, \dots, v_n be the vertices corresponding to second path. Let e_1, e_2, \dots, e_{n-1} be the edges corresponding to first path, $e_n, e_{n+1}, \dots, e_{2n-2}$ be the edges corresponding to second path, $e_{2n-2+i} = u_iv_i$ for $1 \leq i \leq n$ and $e_{3n-2+i} = u_iv_{i+1}$ for $1 \leq i \leq n - 1$. To define $f : E(TL_n) \rightarrow \{0, 1\}$, we consider following four cases.

Case 1: When $n \equiv 0 \pmod{4}$.

$$\begin{aligned}
 f(e_i) &= 0; & 1 \leq i \leq \frac{3n-4}{4}, \\
 f(e_i) &= 1; & \frac{3n}{4} \leq i \leq n-1, \\
 f(e_i) &= 0; & n \leq i \leq \frac{7n-8}{4}, \\
 f(e_i) &= 1; & \frac{7n-4}{4} \leq i \leq 2n-2, \\
 f(e_{2n-1}) &= 0, \\
 f(e_i) &= 1; & 2n \leq i \leq 4n-3.
 \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned}
 v_f(0) &= \frac{3n}{2} \text{ and } v_f(1) = \frac{n}{2} \\
 e_f(0) &= \frac{3n-2}{2} \text{ and } e_f(1) = \frac{5n-4}{2}
 \end{aligned}$$

Thus in this case we have $|(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))| = 1$.

Case 2: When $n \equiv 1 \pmod{4}$.

$$\begin{aligned}
 f(e_i) &= 0; & 1 \leq i \leq \frac{3n-3}{4}, \\
 f(e_i) &= 1; & \frac{3n+1}{4} \leq i \leq n-1, \\
 f(e_i) &= 0; & n \leq i \leq \frac{7n-11}{4}, \\
 f(e_i) &= 1; & \frac{7n-7}{4} \leq i \leq 2n-2, \\
 f(e_{2n-1}) &= 0, \\
 f(e_i) &= 1; & 2n \leq i \leq 4n-3.
 \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned}
 v_f(0) &= \frac{3n-1}{2} \text{ and } v_f(1) = \frac{n+1}{2} \\
 e_f(0) &= \frac{3n-3}{2} \text{ and } e_f(1) = \frac{5n-3}{2}
 \end{aligned}$$

Thus in this case we have $|(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))| = 1$.

Case 3: When $n \equiv 2 \pmod{4}$.

$$\begin{aligned}
 f(e_i) &= 0; & 1 \leq i \leq \frac{3n-2}{4}, \\
 f(e_i) &= 1; & \frac{3n+2}{4} \leq i \leq n-1, \\
 f(e_i) &= 0; & n \leq i \leq \frac{7n-10}{4}, \\
 f(e_i) &= 1; & \frac{7n-6}{4} \leq i \leq 2n-2, \\
 f(e_{2n-1}) &= 0, \\
 f(e_i) &= 1; & 2n \leq i \leq 4n-3.
 \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned}
 v_f(0) &= \frac{3n}{2} \text{ and } v_f(1) = \frac{n}{2} \\
 e_f(0) &= \frac{3n-2}{2} \text{ and } e_f(1) = \frac{5n-4}{2}
 \end{aligned}$$

Thus in this case we have $|(v_f(0) + e_f(0)) - (v_f(1) - e_f(1))| = 1$.

Case 4: When $n \equiv 3 \pmod{4}$.

$$\begin{aligned} f(e_i) &= 0; & 1 \leq i \leq \frac{3n-5}{4}, \\ f(e_i) &= 1; & \frac{3n-1}{4} \leq i \leq n-1, \\ f(e_i) &= 0; & n \leq i \leq \frac{7n-9}{4}, \\ f(e_i) &= 1; & \frac{7n-5}{4} \leq i \leq 2n-2, \\ f(e_{2n-1}) &= 0, \\ f(e_i) &= 1; & 2n \leq i \leq 4n-3. \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned} v_f(0) &= \frac{3n-1}{2} \text{ and } v_f(1) = \frac{n+1}{2} \\ e_f(0) &= \frac{3n-3}{2} \text{ and } e_f(1) = \frac{5n-3}{2} \end{aligned}$$

Thus in this case we have $|(v_f(0) + e_f(0)) - (v_f(1) - e_f(1))| = 1$.

Hence, the triangular ladder TL_n is a total edge product cordial graph. ■

Illustration 2.11. The triangular ladder TL_8 and its total edge product cordial labeling is shown in Figure 5.

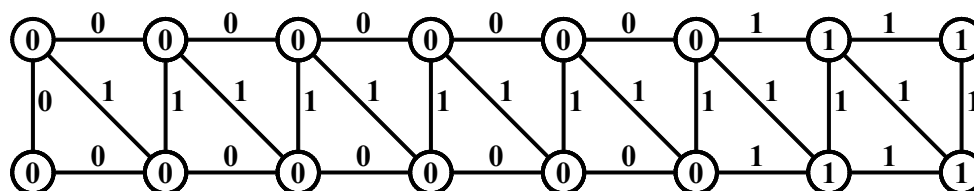


Figure 5

Theorem 2.12. The comb $P_n \odot K_1$ is an edge product cordial graph.

Proof: The comb $P_n \odot K_1$ is of order $2n$ and size $2n-1$. Let v_1, v_2, \dots, v_n are the vertices corresponding to path P_n and $v_{n+1}, v_{n+2}, \dots, v_{2n}$ are the vertices corresponding to K_1 . Let e_1, e_2, \dots, e_{n-1} be the edges corresponding to path P_n and $e_n, e_{n+1}, \dots, e_{2n-1}$ are the remaining edges. We define $f : E(P_n \odot K_1) \rightarrow \{0, 1\}$ as follows.

$$\begin{aligned} f(e_i) &= 0; & 1 \leq i \leq n-1, \\ f(e_i) &= 1; & n \leq i \leq 2n-1. \end{aligned}$$

In view of the above defined labeling pattern we have

$$\begin{aligned} v_f(0) &= v_f(1) = n \\ e_f(0) &= n-1 \text{ and } e_f(1) = n \end{aligned}$$

Thus, we have $|v_f(0) - v_f(1)| = 0$ and $|e_f(0) - e_f(1)| = 1$.

Hence, the comb $P_n \odot K_1$ is an edge product cordial graph. ■

Illustration 2.13. The comb $P_7 \odot K_1$ and its edge product cordial labeling is shown in Figure 6.

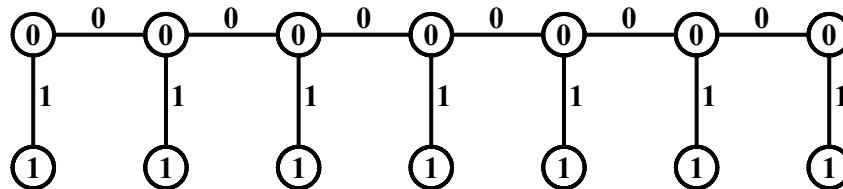


Figure 6

Corollary 2.14. The comb $P_n \odot K_1$ is a total edge product cordial graph.

Proof: The comb $P_n \odot K_1$ is of even order and it is edge product cordial as proved in Theorem 2.12. Then by Proposition 1.4 the result holds. ■

3 Concluding Remarks

Cordial and edge product cordial labeling of a graph are two independent concepts and a graph may possess one or both of these labelings or neither as exhibited below.

1. Every tree is cordial as well as edge product cordial.
2. The friendship graph $C_3^{(t)}$ for $t \equiv 2 \pmod{4}$ is not cordial but it is edge product cordial.
3. The complete bipartite graph $K_{m,n}$ for $m, n \geq 2$ is cordial but not edge product cordial.
4. The complete graph K_n for $n \geq 4$ is neither cordial nor edge product cordial.

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